temporal-logic

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Version 1.0

Abstract

The *temporal-logic* package defines functions for rendering temporal operators defined in *Linear Temporal Logic* (LTL)[1], *Metric Temporal Logic* (MTL)[2], *Metric First-order Temporal Logic* (MFOTL)[3], and the *Counting Metric First-order Temporal Binding Logic* (CMFTBL)[4]. The package defines various functions with variants in order to include or omit optional parameters of the operators like the optional interval.

[1] Pnueli, A. (1977). The temporal logic of programs. In: 18th Annual Symposium on Foundations of Computer Science (SFCS 1977). IEEE. https://doi.org/10.1109/SFCS.1977.32.

[2] Alur, R., Henzinger, T. A. (1993). Real-time logics: Complexity and expressiveness. In: Proceedings of the Fifth Annual Symposium on Logic in Computer Science (LICS 1990). Elsevier. https://doi.org/10.1006/inco.1993.1025.

[3] Basin, David, Klaedtke, Felix, Müller, Samuel, and Zălinescu, Eugen. (2015). Monitoring Metric First-order Temporal Properties. In: Journal of the ACM (J. ACM). Association for Computing Machinery. https://doi.org/10.1145/2699444.

[4] Schallau, T., Naujokat, S., Kullmann, F., Howar, F. (2024). Tree-Based Scenario Classification. In: NASA Formal Methods (NFM 2024). Lecture Notes in Computer Science, vol 14627. Springer, Cham. https://doi.org/10.1007/978-3-031-60698-4_15.

Contents

1 Introduction

1.1 Counting Metric First-order Temporal Logic

Counting Metric First-order Temporal Logic (CMFTBL) argues about finite traces of states. It is an extension of *Metric First-order Temporal Logic* (MFOTL), which itself is based on *Metric Temporal Logic* (MTL) and ultimately on *Linear Temporal Logic* (LTL). All these logics extend the base with further operators and features.

1.2 Basic usage

This package defines the symbols used in Temporal logics as *MathOperators*. Therefore, the symbols, as well as the commands, have to be used in math mode. To use normal text in the parameters `\mathrm` or `\text` may be used to switch back to text mode.

The symbols come in two variants. A standalone version for mentioning logic symbols in text without additional formula spacing is described in Sect. 4 and a formula version, also providing additional parameters, is described in Sect. 2.1 – Sect. 2.5. The formula version should always be preferred over the standalone symbols in formulas as they provide additional spaces and explicitly enforce the correct usage of superscript and subscript via mandatory parameters. The naming scheme of the operators is chosen such that each command reflects the name of the temporal operator prefixed with a "c" for *CMFTBL*. Standalone symbols, as described in Sect. 4, are prefixed with "csymb". Those prefixes are necessary to prevent name clashes with built-in LaTeXcommands.

1.3 Manual structure

This manual is structured as follows. First, the symbols for *Future LTL* and *Past LTL* are introducted in Sect. 2.1 and Sect. 2.2. The additional intervals from *MTL* are described in Sect. 2.3. The operators introduced in *MFOTL* and *CMFTBL* are described in Sect. 2.4 and Sect. 2.5. Section 3 shows the usage of the symbols in formulas. Section 4 closes the command definitions with the standalone symbols for usage in text. Finally, Sect. 5 demonstrates the automatic scaling of the operators.

1.4 Dependencies

The package loads the following dependencies:

- *expl3* For LaTeX3 support.
- *xparse* For parsing the mandatory and optional arguments.
- *amsmath* For symbol definitions.
- *tikz* For rendering of symbols.

2 Symbol definitions

2.1 Future LTL symbols

Future LTL, or simply *LTL*, defines operators to argue about the future. This includes the following operators.

`\ceventually`	`\ceventually`	$\Diamond$
`\cglobally`	`\cglobally`	$\Box$
`\cnext`	`\cnext`	$\bigcirc$
`\cuntil`	`\cuntil`	$\mathcal{U}$

The semantics of the operators are commonly defined as follows:

- $\Diamond\ \phi$ expresses that ϕ must hold at least once in the future.
- $\Box\ \phi$ expresses that ϕ must hold from now for the entire trace.
- $\bigcirc\ \phi$ expresses that ϕ must hold at the next state.
- $\phi\ \mathcal{U}\ \psi$ expresses that ϕ must hold until ψ holds.

2.2 Past LTL symbols

Past LTL defines operators analogous to *Future LTL* to argue about the past. The symbols are identical but are filled solid black.

`\conce`	`\conce`	$\blacklozenge$
`\chistorically`	`\chistorically`	$\blacksquare$
`\cprevious`	`\cprevious`	$\bullet$
`\csince`	`\csince`	$\mathcal{S}$

The semantics of the operators are commonly defined as follows:

- $\blacklozenge\ \phi$ expresses that ϕ must have held at least once in the past.
- $\blacksquare\ \phi$ expresses that ϕ must have held until now for the entire past trace.
- $\bullet\ \phi$ expresses that ϕ must have held at the previous state.
- $\phi\ \mathcal{S}\ \psi$ expresses that ϕ must have held since ψ has held.

2.3 MTL extension

The *Future LTL* and *Past LTL* operators may be extended with an optional interval to form *MTL*.

`\ceventually`	`\ceventually[`⟨*Interval*⟩`]`	$\Diamond_{[0,1]}$
`\conce`	`\conce[`⟨*Interval*⟩`]`	$\blacklozenge_{[0,1]}$
`\cglobally`	`\cglobally[`⟨*Interval*⟩`]`	$\Box_{[0,1]}$
`\chistorically`	`\chistorically[`⟨*Interval*⟩`]`	$\blacksquare_{[0,1]}$
`\cnext`	`\cnext[`⟨*Interval*⟩`]`	$\bigcirc_{[0,1]}$
`\cprevious`	`\cprevious[`⟨*Interval*⟩`]`	$\bullet_{[0,1]}$
`\cuntil`	`\cuntil[`⟨*Interval*⟩`]`	$\mathcal{U}_{[0,1]}$
`\csince`	`\csince[`⟨*Interval*⟩`]`	$\mathcal{S}_{[0,1]}$

The semantics of the intervals are commonly defined as follows: The trace is only evaluated in the given interval. An empty interval is considered to be $[0, \infty)$ for future and $(\infty, 0]$ for past operators. The first component of the interval always indicates the earlier state for both future and past operators.

2.4 MFOTL extension

MFOTL introduces the first-order quantifiers $\exists$ and $\forall$. This package does not provide additional symbols, as the built-in ones already contained in LaTeX may be used.

`\exists`	`\exists`	$\exists$
`\forall`	`\forall`	$\forall$

2.5 CMFTBL extension

CMFTBL extends *MFOTL* by the operators *minPrevalence*, *maxPrevalence*, their past forms, and the *bind* operator.

`\cminprevalence`	`\cminprevalence{⟨Percentage⟩}[⟨Interval⟩]`	$\triangledown^{0.8}_{[0,1]}$
`\cpastminprevalence`	`\cpastminprevalence{⟨Percentage⟩}[⟨Interval⟩]`	$\blacktriangledown^{0.8}_{[0,1]}$
`\cmaxprevalence`	`\cmaxprevalence{⟨Percentage⟩}[⟨Interval⟩]`	$\triangle^{0.8}_{[0,1]}$
`\cpastmaxprevalence`	`\cpastmaxprevalence{⟨Percentage⟩}[⟨Interval⟩]`	$\blacktriangle^{0.8}_{[0,1]}$
`\cbind`	`\cbind{⟨Valuation⟩}{⟨Variable⟩}`	$\downarrow^{v.id}_{i}$

The semantics of the operators are defined as follows. Let p be a fraction of states in the interval $[0, 1]$:

- $\triangledown^{p}_{I}\ \phi$ expresses that ϕ must hold in at least fraction p of the future states in the interval I.
- $\blacktriangledown^{p}_{I}\ \phi$ expresses that ϕ must have held in at least fraction p of the past states in the interval I.
- $\triangle^{p}_{I}\ \phi$ expresses that ϕ must hold in at most fraction p of the future states in the interval I.
- $\blacktriangle^{p}_{I}\ \phi$ expresses that ϕ must have held in at most fraction p of the past states in the interval I.
- $\downarrow^{v.id}_{i}$ saves the valuation $v.id$ to the variable i for later use in a nested formula, where v already has a new value.

minPrevalence and *maxPrevalence* take the desired precentage as another mandatory parameter. These operators may only be defined on finite traces since they argue about numbers of states. *bind* has no optional interval but two mandatory arguments: the value to bind and the target variable.

3 Usage in formulas

The commands may be directly used in math mode to create composite formulas. For the unary formulas, the term ϕ should directly follow the symbol:

`\ceventually\phi`	$\Diamond\ \phi$
`\cglobally[[0,1]]\phi`	$\Box_{[0,1]}\ \phi$

The binary symbols *until* and *since* should be used with two formulas ϕ and ψ directly before and after the symbol:

`\phi\cuntil\psi`	$\phi\ \mathcal{U}\ \psi$
`\phi\csince[[0,1]]\psi`	$\phi\ \mathcal{S}_{[0,1]}\ \psi$

The new *CMFTBL* operators my be used as the unary ones:

`\cminprevalence{0.8}\phi`	$\triangledown^{0.8}\ \phi$
`\cmaxprevalence{0.8}[[0,1]]\phi`	$\triangle^{0.8}_{[0,1]}\ \phi$
`\cbind{v.id}{i}\phi`	$\downarrow^{v.id}_{i}\ \phi$

4 Standalone symbols

The package defines all symbols as a standalone version as *MathOperators* without additional spacing around for the usage in text.

\csymbeventually	\csymbeventually	◊
\csymbonce	\csymbonce	⧫
\csymbglobally	\csymbglobally	□
\csymbhistorically	\csymbhistorically	■
\csymbnext	\csymbnext	○
\csymbprevious	\csymbprevious	●
\csymbuntil	\csymbuntil	$\mathcal{U}$
\csymbsince	\csymbsince	$\mathcal{S}$
\csymbminprevalence	\csymbminprevalence	▽
\csymbpastminprevalence	\csymbpastminprevalence	▼
\csymbmaxprevalence	\csymbmaxprevalence	△
\csymbpastmaxprevalence	\csymbpastmaxprevalence	▲
\csymbbind	\csymbbind	↓

5 Operator scaling

The operators scale automatically with the current text size:

\Huge ◊ [0,1] ϕ

\huge ◊ [0,1] ϕ

\LARGE ◊ [0,1] ϕ

\Large ◊ [0,1] ϕ

\large ◊ [0,1] ϕ

\normalsize ◊ [0,1] ϕ

\small ◊ [0,1] ϕ

\footnotesize ◊ [0,1] ϕ

\scriptsize ◊ [0,1] ϕ

\tiny ◊ [0,1] ϕ

6 License

The current maintainer of this work is
Dominik Schmid <dominik.schmid@tu-dortmund.de>.

This work consists of the files temporal-logic.dtx, temporal-logic.ins,
and the derived file temporal-logic.sty.

7 Sourcecode

```
% <*package>
% <@@=cmftbl>

\RequirePackage{amsmath}
\RequirePackage{expl3}
\RequirePackage{xparse}
\RequirePackage{tikz}

\ProvidesExplPackage {temporal-logic} { 2024-10-17 } { v1.0 }{
  Symbols for Temporal Logics
}

\cs_new:Nn \__cmftbl_op_sup_sub:Nnn {
      \ensuremath {
        #1
        \tl_if_empty:nF { #2 } { \c_math_superscript_token { \,#2 } }
        \tl_if_empty:nF { #3 } { \c_math_subscript_token { \,#3 } }
        \,
      }
}
\cs_generate_variant:Nn \__cmftbl_op_sup_sub:Nnn { cnn }

\cs_new:Nn \__cmftbl_op_sup:Nn { \__cmftbl_op_sup_sub:Nnn { #1 } { #2 } {} }
\cs_generate_variant:Nn \__cmftbl_op_sup:Nn { cn }

\cs_new:Nn \__cmftbl_op_sub:Nn { \__cmftbl_op_sup_sub:Nnn { #1 } { } { #2 } }
\cs_generate_variant:Nn \__cmftbl_op_sub:Nn { cn }

\cs_new:Nn \__cmftbl_op:N { \__cmftbl_op_sup_sub:Nnn { #1 } { } { } }
\cs_generate_variant:Nn \__cmftbl_op:N { c }
```

```
\dim_new:N \__cmftbl_fht_dim
\cs_new:Nn \__cmftbl_ex: { \dim_use:N \__cmftbl_fht_dim }

\cs_new:Nn \__cmftbl_render_op:n {
  \dim_set:Nn \__cmftbl_fht_dim {\fontcharht\font`X}
  \tikz[execute~at~end~picture={
       \useasboundingbox (0, 0) rectangle (\__cmftbl_ex:, \__cmftbl_ex:);
  }]{
    \group_begin:
      \cs_set_eq:NN \EX \__cmftbl_ex:
      #1
    \group_end:
  }
}

\DeclareMathOperator { \csymbeventually } {
  \__cmftbl_render_op:n {
     \draw
       (.5*\EX, 0) --
       (.2*\EX, .5*\EX) --
       (.5*\EX, \EX) --
       (.8*\EX, .5*\EX) --
       cycle;
  }
}
\DeclareMathOperator { \csymbonce } {
  \__cmftbl_render_op:n {
    \draw[fill]
    (.5*\EX, 0) --
    (.2*\EX, .5*\EX) --
    (.5*\EX, \EX) --
    (.8*\EX, .5*\EX) --
    cycle;
}
}
\DeclareMathOperator { \csymbglobally } {
  \__cmftbl_render_op:n {
    \draw
    (.15*\EX, .15*\EX)
    rectangle
    (.85*\EX, .85*\EX);
  }
}
\DeclareMathOperator { \csymbhistorically } {
  \__cmftbl_render_op:n {
    \draw[fill]
    (.15*\EX, .15*\EX)
    rectangle
    (.85*\EX, .85*\EX);
  }
}
\DeclareMathOperator { \csymbnext } {
  \__cmftbl_render_op:n {
```

```
    \draw
    (.5*\EX, .5*\EX)
    circle
    (.4*\EX);
  }
}
\DeclareMathOperator { \csymbprevious } {
  \__cmftbl_render_op:n {
    \draw[fill]
    (.5*\EX, .5*\EX)
    circle
    (.4*\EX);
  }
}
\DeclareMathOperator { \csymbuntil } {
  \ensuremath\mathcal{U}
}
\DeclareMathOperator { \csymbsince } {
  \ensuremath\mathcal{S}
}
\DeclareMathOperator { \csymbminprevalence } {
  \__cmftbl_render_op:n {
    \draw
      (.1*\EX, .9*\EX) --
      (.9*\EX, .9*\EX) --
      (.5*\EX, .1*\EX) --
      cycle;
  }
}
\DeclareMathOperator { \csymbpastminprevalence } {
  \__cmftbl_render_op:n {
    \draw[fill]
    (.1*\EX, .9*\EX) --
    (.9*\EX, .9*\EX) --
    (.5*\EX, .1*\EX) --
    cycle;
  }
}
\DeclareMathOperator { \csymbmaxprevalence } {
  \__cmftbl_render_op:n {
    \draw
    (.1*\EX, .1*\EX) --
    (.9*\EX, .1*\EX) --
    (.5*\EX, .9*\EX) --
    cycle;
}
  }
\DeclareMathOperator { \csymbpastmaxprevalence } {
  \__cmftbl_render_op:n {
    \draw[fill]
    (.1*\EX, .1*\EX) --
    (.9*\EX, .1*\EX) --
    (.5*\EX, .9*\EX) --
    cycle;
```

```
  }
}
\DeclareMathOperator { \csymbbind } {
  \__cmftbl_render_op:n {
    \draw (.5*\EX, \EX) -- (.5*\EX, 0);
    \draw
      (.2*\EX, .3*\EX) ..
      controls (.4*\EX, .2*\EX) ..
      (.5*\EX, 0) ..
      controls (.6*\EX, .2*\EX) ..
      (.8*\EX, .3*\EX);
  }
}

\ProvideDocumentCommand { \ceventually } { O{} } {
  \__cmftbl_op_sub:cn { csymbeventually } { #1 }
}
\ProvideDocumentCommand { \conce } { O{} } {
  \__cmftbl_op_sub:cn { csymbonce } { #1 }
}
\ProvideDocumentCommand { \cglobally } { O{} } {
  \__cmftbl_op_sub:cn { csymbglobally } { #1 }
}
\ProvideDocumentCommand { \chistorically } { O{} } {
  \__cmftbl_op_sub:cn { csymbhistorically } { #1 }
}
\ProvideDocumentCommand { \cnext } { O{} } {
  \__cmftbl_op_sub:cn { csymbnext } { #1 }
}
\ProvideDocumentCommand { \cprevious } { O{} } {
  \__cmftbl_op_sub:cn { csymbprevious } { #1 }
}
\ProvideDocumentCommand { \cuntil } { O{} } {
  \__cmftbl_op_sub:Nn { \;\csymbuntil } { #1 }
}
\ProvideDocumentCommand { \csince } { O{} } {
  \__cmftbl_op_sub:Nn { \;\csymbsince } { #1 }
}
\ProvideDocumentCommand { \cminprevalence } { m O{} } {
  \__cmftbl_op_sup_sub:cnn { csymbminprevalence } { #1 } { #2 }
}
\ProvideDocumentCommand { \cpastminprevalence } { m O{} } {
  \__cmftbl_op_sup_sub:cnn { csymbpastminprevalence } { #1 } { #2 }
}
\ProvideDocumentCommand { \cmaxprevalence } { m O{} } {
  \__cmftbl_op_sup_sub:cnn { csymbmaxprevalence } { #1 } { #2 }
}
\ProvideDocumentCommand { \cpastmaxprevalence } { m O{} } {
  \__cmftbl_op_sup_sub:cnn { csymbpastmaxprevalence } { #1 } { #2 }
}
\ProvideDocumentCommand { \cbind } { m m } {
  \__cmftbl_op_sup_sub:cnn { csymbbind } { #1 } { #2 }
}
% </package>
```